Hysteresis in isotropic spin systems

1993 J. Phys. A: Math. Gen. 26 3973

(http://iopscience.iop.org/0305-4470/26/16/014)

J. Phys. A: Math. Gen. 26 (1993) 3973–3981. Printed in the UK

Hysteresis in isotropic spin systems

Peter B Thomas† and Deepak Dhar‡

Theoretical Physics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India

Received 21 December 1991, in final form 18 May 1993

Abstract. We consider hysteresis in isotropic N-vector models in d dimensions in an external spatially uniform field varying sinusoidally in time. We use renormalization group arguments to show that for $d > 2$ and $N \geqslant 2$, for small frequencies ω, and small amplitudes H_0 of the field, the area of the hysteresis loop scales as $(H_0\omega)^{1/2}$ with logarithmic corrections. For $N = 1$ and $d > 1$, using nucleation theory we show that the area for $\omega \ll H_0$ scales as $|T \ln(H_0\omega)|^{-1/(d-1)}$. The power-law dependence of the area of hysteresis loops in continuous spin systems is a manifestation of their self-organized criticality.

1. Introduction

In recent years there has been much interest in the power laws seen in open systems driven out of equilibrium [1]. Magnetic hysteresis is a simple example of a system in contact with a thermal reservoir, driven by an external time-dependent field. A brief review of some of the experimental properties of hysteresis in magnets can be found in [2]. Although it is a very familiar phenomenon, theoretical attempts at understanding it have so far been mainly phenomenological [3]. Recently, evolution equations for the order parameter in various driven systems in the mean-field approximation have been discussed [4,5]. There are also some numerical studies mainly in two-dimensional Ising-like systems using Glauber and cell dynamics [6,7]. Sethna *et al* have studied the zero-temperature dynamics of the random-field Ising model, and shown that this simple model shows the so-called return point memory effect and Barkhausen-noise-like fluctuations in its hysteretic response [8]. In continuous spin systems, relaxation in the N-vector model in the limit $N \to \infty$ starting from the Langevin equation has been discussed by Mazenko and Zannetti [9] and by Rao *et al* [6, 10]. Based on numerical evidence, Rao *et al* have suggested that the area of the hysteresis loops has a power-law dependence on the frequency and amplitude of the external magnetic field.

In an earlier work [11], we have studied hysteresis in the N-vector model in the limit $N \to \infty$. In this limit we showed that, for small magnitudes of the external magnetic field H_0 and at sufficiently low frequencies ω, the area of the hysteresis loop in the model varies as $(H_0\omega)^{1/2}$ below the critical temperature, in all dimensions $d > 2$ (the lower critical dimension). The same result has also been obtained by Somoza and Desai [12] using a very similar approach. The case $N = 2$, $d = 2$ is special, and shows a continuously varying exponent with temperature, at all temperatures below the Kosterlitz–Thouless transition temperature [13]. In this paper, we use renormalization arguments to generalize this result

† e-mail address: peter@tifrvax.bitnet

‡ e-mail address: ddhar@tifrvax.bitnet

to all $N \geqslant 2$ models with local ferromagnetic interactions and isotropic exchange couplings. We also show, using nucleation arguments, that for the $N = 1$ case (Ising model) the frequency dependence of the area is much weaker, and it varies as $|T \ln(H_0\omega)|^{-1/(d-1)}$ in dimensions $d > 1$, in the limit of small ω.

We first consider the case $N \geqslant 2$. For definiteness, we consider a system of planar spins (the XY model) on a d-dimensional hypercubic lattice interacting with each other by nearest-neighbour ferromagnetic couplings which are *isotropic in the spin space*. Let θ_i ($0 \leqslant \theta_i \leqslant 2\pi$) be the spin variable on each lattice site i. The Hamiltonian of the system is then given by

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) - H(t) \sum_i \cos\theta_i \tag{1.1}$$

where $\langle ij \rangle$ denotes the sum over all nearest-neighbour pairs. The system is in contact with a heat bath at temperature T, less than the critical temperature T_c. Note that we do not take into account dipolar forces. These give rise to domain formation in real ferromagnets, and a significant part of the hysteretic response of a multi-domain magnetic sample comes from the motion of domain walls (for a discussion, see for example, [14]). Our treatment may be expected to be applicable to single-domain magnetic grains with small crystalline anisotropy. We also ignore precession and assume that each spin evolves under Langevin dynamics, the evolution equations in suitable units of time being given by

$$\frac{\mathrm{d}\theta_i}{\mathrm{d}t} = -J \sum_j \sin(\theta_i - \theta_j) - H_0 \sin\omega t \sin\theta_i + \eta_i(t) \tag{1.2}$$

where j are all the nearest-neighbour sites to i. Here $\eta_i(t)$ is Gaussian white noise of zero mean, uncorrelated at different sites i. The variance of $\eta_i(t)$ is related to the temperature T of the heat bath, as usual.

In the absence of a magnetic field, since $T < T_c$, there exists a spontaneous magnetization in thermal equilibrium. The fluctuations from the reference ground state are of two kinds: spin waves and topological excitations or defects (vortices or vortex lines for $N = 2$, hedgehogs for $N = 3$, etc). The defects are known to drive the order–disorder transition at least for small N. However, a uniform time-dependent field tends to turn all the spins similarly, and thus does not lead to a significant change in the density of defects. At low temperatures, for not too small fields, there are only a few tightly bound defect pairs in equilibrium. A small density of defect pairs is created at domain walls for brief periods during magnetization reversals. But these annihilate very soon after, and do not significantly affect the overall magnetization. Thus, in the context of hysteresis, it is sufficient to consider only the nonlinear dynamics of spin waves.

For low temperatures the equilibrium properties are known to be well described by the spin-wave approximation. The nonlinearities giving rise to interaction between the spin-waves lead only to a small renormalization of the spin-wave energy spectrum, but the qualitative behaviour of spin correlations at large distances is *not* affected by small nonlinearities. We thus expect that the spin-wave analysis suitably adapted will also provide a correct description in the non-equilibrium situation.

2. Spin-wave approximation for large k modes

The set of equations (1.2) are hard to solve analytically, because an infinite hierarchy of coupled differential equations relating the expectation values of the various correlation functions (the BBGKY hierarchy) is obtained. In this section we describe an approximation scheme that distinguishes between the short and long wavelength fluctuations. The short-distance fluctuations are roughly in instantaneous thermal equilibrium at all times, and are quite well described by the non-interacting spin-wave approximation. However, it is not possible to treat the long-wavelength fluctuations (whose amplitudes are much larger) in the same way, because of the nonlinearities present in the system. These are much better described in *real* space by assuming that the evolution at sites very far away from each other are roughly independent. The value of the wavenumber k (denoted by k^* in the following) which separates the spin-waves and the local fluctuation regimes is determined self-consistently. Thus while the large k modes are treated in the linear spin-wave approximation, we are able to determine the *nonlinear* hysteretic response of the system by correctly taking into account the nonlinear evolution of localized block spins.

In the static case, the transverse correlation length is infinite only in zero field. In the presence of a time-varying field there is a significant transverse component of the spin–spin correlation functions (particularly during magnetization reversals). However, even in the presence of time-varying fields the connected part of the correlation function in the steady state *is finite ranged at all times*. As the magnitude of $H(t)$ decreases, the correlation length starts to increase, but at finite frequencies the magnitude of the field starts increasing long before it could become infinite. If the maximum value of the (time-dependent) correlation length is $L^*(H_0, \omega)$, at *no* time is the transverse components of the spins significantly correlated for distances greater than L^*. Keeping L^* a yet undetermined parameter, we make a spin-wave analysis of the fluctuations not about a uniformly magnetized state with long-ranged correlations, but about a state which is (possibly) magnetized, but in which connected correlation functions extend, at most, to a distance L^*. We denote by θ_k the Fourier transform of θ_i, and express θ_i as a sum of a slowly varying function $\overline{\theta}_i$ (the average over separations of order L^*), and short-distance fluctuations $\delta\theta_i$

$$\theta_i = \overline{\theta}_i + \delta\theta_i \tag{2.1}$$

where the Fourier decomposition of $\overline{\theta}_i$ has only modes $k < k^* \equiv \pi/L^*$, and $\delta\theta_i$ has only Fourier components with $k > k^*$.

As $\langle(\delta\theta_i - \delta\theta_{i+\delta})^2\rangle$ decreases as T, for small temperatures T of the system, one can replace $\sin(\theta_i - \theta_{i+\delta})$ by its linear approximant $(\theta_i - \theta_{i+\delta})$. We then obtain from (1.2), the equation of motion of $\delta\theta_k$, the amplitude of the short-wavelength fluctuations of wavenumber k.

$$\frac{\mathrm{d}\delta\theta_k}{\mathrm{d}t} = -2J\sum_{\mu=1}^{d}(1-\cos k_\mu)\delta\theta_k - H_0\sin\omega t\frac{1}{\sqrt{N}}\sum_j \cos\overline{\theta}_j\,\delta\theta_j \mathrm{e}^{-\mathrm{i}k\cdot j} + \eta_k \qquad \text{for } k > k^*. \tag{2.2}$$

The response of the system to a slowly varying field is determined by the long wavelength fluctuations (the small k modes) in the system so, for simplicity, we can use the approximation $(1 - \cos k_\mu) \sim k_\mu^2/2$, which makes the spin-wave spectrum purely quadratic in k, and spherically symmetric. At all times except during magnetization reversals, the spins are aligned in the direction of the field and so it is reasonable to replace $\cos\overline{\theta}_i$ by $m(t)$, the magnetization at time t. This is equivalent to a linearization approximation for the equation (2.2). Let $\sigma_k^2(t)$ denote the expectation value of $|\delta\theta_k|^2$. It is easy to see that it satisfies the equation

$$\frac{\mathrm{d}\sigma_k^2}{\mathrm{d}t} = 2T - 2(Jk^2 + m(t)H_0\sin\omega t)\sigma_k^2. \tag{2.3}$$

From this one can estimate the maximum correlation length L^* using the condition that the small wavelength fluctuations ($k > k^*$) are in approximate instantaneous thermal equilibrium at all times, while for the modes with $k < k^*$, hysteretic effects cannot be neglected. If the modes with wavenumber k are approximately in instantaneous thermal equilibrium, we get

$$\sigma_k^2 \approx \frac{T}{Jk^2 + H(t)m(t)}. \tag{2.4}$$

We note that the functional dependence in this equation is slightly different from the exact result for the N-vector $(\phi \cdot \phi)^2$ model in the $N \to \infty$ limit [9], the hysteretic behaviour of which we have discussed in [11]. In this limit,

$$\sigma_k^2 = \frac{T}{k^2 + H(t)/m(t)}.$$

However, since we are working at temperatures well below the critical temperature and the system has finite spontaneous magnetization $m \approx m_0 \approx 1$, this is not a serious difference.

We have argued that the large k modes are in approximate instantaneous thermal equilibrium, and have a time dependence roughly similar to that in (2.4). In order to find the characteristic value of k above which (2.4) is approximately true, we note that by equation (2.3), whenever the magnetization $m(t)$ has the same sign as the field $H(t)$, the magnitude of $\mathrm{d}\sigma_k^2/\mathrm{d}t$ is less than $2T$. For small $H(t)$, $\mathrm{d}H/\mathrm{d}t \approx H_0\omega$ and equation (2.3) is consistent with equation (2.4), provided

$$k^4 \geqslant H_0\omega/2J^2. \tag{2.5}$$

We may thus identify the cross-over value k^* with $[H_0\omega/2J^2]^{1/4}$. As the amplitude or the frequency of the field tends to zero, k^* can be made arbitrarily small. This is consistent with the known non-exponential decay of the transverse correlation function in zero field. We note that this identification of k^* is in agreement with our ealier result for the limit $N \to \infty$, in which case a saddle-point analysis of the equations of motion shows that $k^* \propto (H_0\omega)^{1/4}$. For modes with $k < k^*$ equation (2.3) may be simplified as

$$\frac{\mathrm{d}\sigma_k^2}{\mathrm{d}t} \simeq 2T - 2H(t)m(t)\sigma_k^2 \qquad \text{for } k < k^* \tag{2.6}$$

and is roughly independent of k at all times t. This is equivalent to assuming that the spin field $\overline{\theta}$ is essentially uncorrelated for length scales $L \geqslant L^*$. Since modes with $k > k^*$ are approximately in instantaneous thermal equilibrium at all times, we may integrate these degrees of freedom away and consider the evolution of $\overline{\theta}(x)$ field averaged over length scale L^*. The equations of motion from equation (1.2) are again of the form

$$\frac{\mathrm{d}\overline{\theta}}{\mathrm{d}t} \approx \tilde{J}\nabla^2\overline{\theta} - \tilde{H}(t)\ \sin\overline{\theta} + \tilde{\eta}(t) \tag{2.7}$$

with renormalized couplings

$$\tilde{J} \simeq JL^{*-2} \qquad \tilde{H}(t) \simeq H(t)\mathrm{e}^{-\sigma^2(t)/2} \qquad \tilde{T} \simeq TL^{*-d} \tag{2.8}$$

and $\sigma^2(t) = (1/\sqrt{N})\sum_{k>k^*}\sigma_k^2(t)$ with $\sigma_k^2(t)$ given by equation (2.3).

The renormalized coupling constants will differ from these values if the nonlinear interactions between spin-waves is taken into account. However, it is clear from equation (2.8) that the response of the system is determined by the zero-temperature fixed point (provided $T < T_c$) and, therefore, *the nonlinear spin-wave couplings are irrelevant.* Hence the linear spin-wave analysis is sufficient to determine the exponents describing the power law for the area of the loops.

If L^* is large (small frequencies, small fields), then $\tilde{J}$ is much smaller than $\tilde{H}$ and can be neglected. Then the hysteresis phenomena can be described in terms of *independent, uncoupled, effective block spins.* The Fokker–Planck equation of the probability distribution of $\overline{\theta}$, $P(\overline{\theta}, t)$ is†

$$\frac{\partial P}{\partial t} = \tilde{H}(t)\frac{\partial}{\partial\overline{\theta}}(P\sin\overline{\theta}) + \tilde{T}\frac{\partial^2 P}{\partial\overline{\theta}^2}. \tag{2.9}$$

Consistent with our linear approximation, the large k fluctuations are assumed to be distributed normally about $\overline{\theta}$, and the magnetization is therefore given by

$$m(t) = \mathrm{e}^{-\sigma^2(t)/2}\int_0^{2\pi} P(\overline{\theta}, t)\cos\overline{\theta}\,\mathrm{d}\overline{\theta}. \tag{2.10}$$

Equations (2.3), (2.8) and (2.9), provide a full description of hysteresis at low frequencies, in our approximation scheme.

At small frequencies and fields, one can find the limiting scaling form of the area analytically, as follows. If the field H_0 is small, then for large k one has $\sigma_k^2(t) \sim T/k^2$, which is approximately independent of time. Therefore in this limit, $\tilde{H}(t) \sim \tilde{H}_0\sin\omega t$ where $\tilde{H}_0 \sim H_0\mathrm{e}^{-\alpha T}$, and α is a constant. When $\omega t \sim 0$, we can linearize $\tilde{H}(t)$ by

$$\tilde{H}(t) \simeq \mathrm{e}^{-\alpha T} H_0\omega t. \tag{2.11}$$

At low frequencies, L^* is very large and $\tilde{H}_0 \gg \tilde{T}$. Thus most of the time, the probability distribution is very sharply peaked at $\overline{\theta} = 0$ or π. For small $\overline{\theta}$ the Fokker–Planck equation (2.9) can be linearized to give

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial\overline{\theta}}(P\overline{\theta}\tilde{H}_0\omega t) + \tilde{T}\,\frac{\partial^2 P}{\partial\overline{\theta}^2}. \tag{2.12}$$

It is easy to see that the solution of this equation is a Gaussian in $\overline{\theta}$

$$P(\overline{\theta}, t) = \frac{\mathrm{e}^{-\overline{\theta}^2/2\gamma^2(t)}}{\gamma(t)\sqrt{2\pi}}. \tag{2.13}$$

The solution for $\gamma(t)$ consistent with the boundary condition at large negative times is

$$\gamma^2(t) = \frac{\tilde{T}\sqrt{\pi}}{\sqrt{\tilde{H}_0\omega}}\mathrm{e}^{\tilde{H}_0\omega t^2}\left[1 + \mathrm{erf}\left(t\sqrt{\tilde{H}_0\omega}\right)\right]. \tag{2.14}$$

† A similar equation including the effects of anisotropy has been studied earlier by Kumar and Dattagupta to determine the AC susceptibility [15]. However, they studied only the linear response, while our interest is mainly in the nonlinear effects in hysteresis.

As a check, we find that at, as $t \to \infty$, the fluctuations are approximately at their instantaneous equilibrium value, $\gamma^2(t) \sim \tilde{T}/|\tilde{H}(t)|$.

The coercive field H_c is the magnitude of the field when $\gamma^2(t) \sim O(1)$. Since the maximum value of m is its spontaneous magnetization $m_0 \simeq 1$ at low fields, the area $W \sim H_c$, which therefore varies as

$$W \sim (H_0\omega)^{1/2}\{\ln[(H_0\omega)^{(2-d)/4}/T]\}^{1/2}. \tag{2.15}$$

It is interesting to observe that the dependence on H_0 and ω for the area is essentially the same as that for the N-vector model in the $N \to \infty$ limit, including the logarithmic correction [11].

We note that since k^{*2} is of the order of $(H_0\omega)^{1/2}$, when $\omega > H_0$ the inter-block couplings $\tilde{J} > \tilde{H}_0$, and cannot be neglected, so our treatment breaks down at high frequencies. However from a systematic $1/\omega$ perturbation series one can easily see that $W \sim H_0^2/\omega$ for large ω. In the $N \to \infty$ case, there is a dynamical phase transition separating the low-frequency regime from the high-frequency regime [11], in which the time average of the magnetization has a non-zero expectation value, in the direction transverse to the magnetic field. Unfortunately since our approximation scheme is valid only when $\omega \ll H_0$, we cannot address the interesting question as to whether there is also a dynamical phase transition at high frequencies for finite N.

In order to check the validity of the approximate solution (2.11), where $\sigma^2(t)$ was taken to be approximately independent of time (for small H_0), we have numerically solved the equations (2.3) and (2.9) directly, by expanding $P(\overline{\theta}, t)$ in a Fourier series in $\overline{\theta}$. We define $g_n(t)$ by the expansion

$$P(\overline{\theta}, t) = \frac{1}{2\pi}\sum_{n=0}^{\infty} g_n(t)\cos n\overline{\theta}. \tag{2.16}$$

By conservation of probability, g_0 is independent of time, and is normalized to unity, i.e. $g_0 \equiv 1$. Substituting equations (2.16) in (2.14), we get an infinite series of linear, coupled, ordinary differential equations

$$\frac{\mathrm{d}g_n}{\mathrm{d}t} = \frac{n}{2}\tilde{H}_0 \sin\omega t\,(g_{n-1} - g_{n+1}) - n^2\tilde{T} \qquad n \geqslant 1. \tag{2.17}$$

As $P(\overline{\theta}, t)$ is approximately Gaussian in $\overline{\theta}$, the coefficients g_n decay as $\exp[-n^2/2\Delta]$ [see equation (2.13)]. In our numerical work, we kept terms in n up to a maximum value of $n_{\max} = 5\Delta$. As a check, we doubled the value of $n_{\max}$, and found that there was no significant change in the solutions. The equations were solved using a variable-step fourth-order Runge–Kutta method. In figure 1 we compare the the frequency dependence of the area on the product $(H_0\omega)$ over a range of several decades, with the predicted form (2.15). The fit is not very sensitive to the choice of the fitting parameter K. We see that the fit is very good in the regime $\omega \ll H_0$. We thus see that the exact numerical solution of the nonlinear evolution equations (2.3) and (2.9) agrees very well with our analytically derived approximate solution (2.13) and (2.14). A direct check of the validity of equations (2.3) and (2.9) as a good approximation to the Langevin dynamical evolution of the spins was not attempted, as it would require much computer time.

All the above analysis is easily generalized to the $N > 2$ models. There are $(N-1)$ transverse degrees of freedom, each of which evolve according to (2.3), giving a prefactor of $(N-1)$ to $\sigma^2(t)$. The renormalization of J and T is the same as in the case $N = 2$. At low frequencies, the uncoupled block spins will be N-dimensional vectors, but since the time dependence of the probability distribution is only a function of the azimuthal angle θ, the resulting equation is the same as (2.9). Thus the hysteretic behaviour for $T < T_c$ is the same in all $N \geqslant 2$ models.

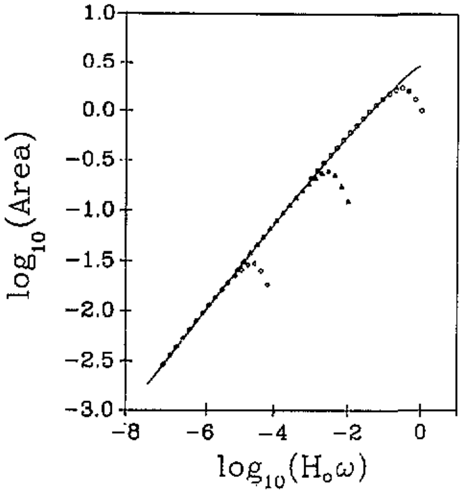

Figure 1. A log–log plot of the area of the hysteresis loop against $(H_0\omega)$ for the continuous spin case $N \geqslant 2$, for $H_0 = 0.01$ (squares), $H_0 = 0.1$ (triangles) and $H_0 = 1$ (circles). The full curve is our theoretical formula (2.15), $W = a(H_0\omega)^{1/2}\{\ln[K/\tilde{H}_0\omega]\}^{1/2}$ for $a \simeq 2.15$ and $K \approx 7.0$.

3. Hysteresis in the Ising case

For $N = 1$ (the Ising model), there are no transverse fluctuations, and this leads to a qualitatively different hysteretic response compared with $N \geqslant 2$. In this case, the magnetization reversals occur by the nucleation mechanism [16]. For small fields H, the size of a critical droplet of up spins, in a sea of down spins varies as $\sigma(T)/H$, where $\sigma(T)$ is the temperature-dependent surface tension energy. Thus the excess energy of a critical droplet $\sim \sigma^d(T)/H^{d-1}$, and the rate of nucleation $\sim \exp[-\sigma^d(T)/TH^{d-1}]$. These predictions of nucleation theory have been numerically verified in two and three dimensions [17]. Once nucleated, these droplets grow, and the rate of increase in the droplet radius is proportional to H. For a very small frequency ω, we can determine the coercive field H_c using the condition that at the corresponding time t_c, the average volume of droplets per site ≈ 1, i.e.

$$(H_c t_c)^d \int_0^{t_c} dt' \exp\left[-k_1\,\sigma^d(T)/T H^{d-1}(t')\right] \simeq k_2$$

where k_1 and k_2 are constants of order unity. Using $t_c = H_c/H_0\omega$, we get

$$H_c^{2d+2} \exp\left[-k_1\,\sigma^d(T)/T H_c^{d-1}\right] \simeq k_2\,(H_0\omega)^{d+1} \tag{3.1}$$

which shows that, in the limit of small frequency, the coercive field and hence the area of the hysteresis loop varies as

$$W \sim |T\ln(H_0\omega)|^{-1/(d-1)}. \tag{3.2}$$

In numerical simulations of hysteresis in Ising-like systems, the dependence of W on H_0 and ω has been fitted to power laws [10, 7]. This is presumably because the values of H_0 and ω used are too large for the asymptotic law (3.2) to be valid, and be significantly distinguishable from a power law.

In order to test the validity of our prediction (3.1), we have studied the variation of W with $(H_0\omega)$ in the single-spin flip Glauber model with sequential updating of spins using Monte Carlo simulations. We performed simulations on a 100×100 square lattice with periodic boundary conditions. The field was assumed to have a linear dependence on time, going from a large negative value to a large positive value. The magnetization loops obtained were quite reproducible. We averaged the magnetization over 20 cycles. In figure 2 we have plotted the variation of W with $(H_0\omega)$ in our simulations for three different temperatures ($T = J$, $5J/4$ and $3J/2$). The theoretical fits using equation (3.1), treating k_1 and k_2 as temperature-dependent fitting parameters, are also plotted. We see that the fits are quite satisfactory. It may also be noted that the slope of the log–log curve changes continuously but slowly, and decreases as $(H_0\omega)$ is decreased. The effective power-law form $W \sim (H_0\omega)^\alpha$ gives a value $\alpha \approx 0.2$ for $(H_0\omega)$ in the range 10^{-4} to 10^{-2} in our units.

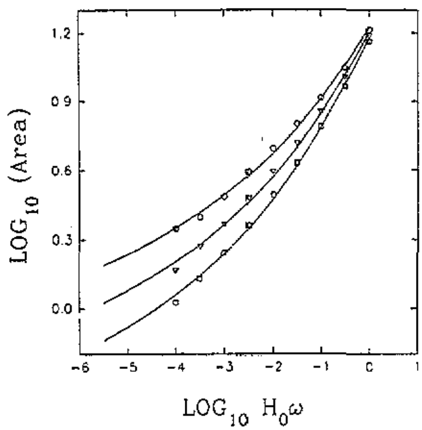

Figure 2. Same as figure 1, for $N = 1$: data for $T = J$ (circles), $T = 5J/4$ (triangles) and $T = 3J/2$ (squares). The full curves are our theoretical predictions treating k_1 and k_2 as temperature-dependent fitting parameters.

4. Summary and discussion

We have shown that in N-vector spin models $N \geqslant 2$ in all dimensions $d > 2$, the area of the hysteresis loops has a universal scaling behaviour $W \sim (H_0\omega)^{1/2}$ at low amplitudes and frequencies of the field. This robust power-law behaviour in the area of the hysteresis loops in these models is a signature of self-organized criticality. We note that in hysteresis we are concerned with the non-equilibrium steady state in a dissipative system where nonlinear effects are important. These are the most important characteristics of self-organizing critical systems.

In all continuous spin models there is self-organized criticality at the first-order transition line $H = 0$, because there are power-law correlations in the transverse fluctuations in the order parameter [18]. In hysteresis, there *is* a non-zero time-dependent driving field. We note that for finite non-zero values of $(H_0\omega)$, the system has a finite correlation length L^*, and is not truly critical. However as we have shown, the exponent characterizing the $(H_0\omega)$

dependence of the area of hysteresis loops depends on the (non-equilibrium) nonlinear response of the critical state of the system at $H = 0$; and may be used to characterize the latter. Hence the power-law dependence of the area of hysteresis loops on H_0 and ω can be thought of as a manifestation of the self-organized criticality of the system in zero field.

Put differently, the significant contribution to the area of the hysteresis loop comes from the time when the system is driven through the two-phase coexistence region. In this case there is no free energy cost associated with changes in the order parameter, and hence no 'bulk' thermodynamical restoring force to such changes. This leads to a gapless spectrum of relaxation rates in the system, which lies at the root of the power-law behaviour in the area of loops. Hysteresis in isotropic continuous spin systems thus provides us with a simple example of self-organized criticality.

References

[1] See, for example, the articles in Godrèche C (ed) 1992 *Solids Far from Equilibrium* (New York: Cambridge University Press)
[2] Smit J and Wijn H P J 1959 *Ferrites* Philips Technical Library, Eindhoven, ch 15 and 16
[3] Preisach E 1935 *Z. Phys.* **94** 277
Stoner E C and Wohlfarth E P 1948 *Phil. Trans. R. Soc.* A **240** 599
Godrèche C (ed) 1992 *Solids Far from Equilibrium* (New York: Cambridge University Press) pp 327–30
[4] Gilmore R 1979 *Phys. Rev.* A **20** 2510
Agarwal G S and Shenoy S R 1981 *Phys. Rev.* A **23** 2719
Shenoy S R and Agarwal G S 1984 *Phys. Rev.* A **29** 1315
[5] Jung P, Gray G and Roy R 1990 *Phys. Rev. Lett.* **65** 1873
Tomé T and de Oliveira M J 1990 *Phys. Rev.* A **41** 4251
[6] Rao M, Krishnamurthy H R and Pandit R 1990 *Phys. Rev.* B **42** 856
[7] Lo W S and Pelcovits R A 1990 *Phys. Rev.* A **42** 7471
Sengupta S, Marathe Y J and Puri S 1992 *Phys. Rev.* B **45** 7828
Acharyya M, Chakrabarti B K and Sen A K 1992 *Physica* A **186** 231; 1993 *Physica* A **192** 471
[8] Sethna J P, Dahmen K, Kartha S, Krumhans J A, Roberts B W and Shore J D 1993 *Phys. Rev. Lett.* **70** 3347
[9] Mazenko G and Zannetti M 1985 *Phys. Rev.* B **32** 4565
[10] Rao M, Krishnamurthy H R and Pandit R 1991 *J. Phys.: Condens. Matter* **1** 9061
[11] Dhar D and Thomas P B 1992 *J. Phys. A: Math. Gen.* **25** 4967
[12] Somoza A M and Desai R C 1993 *Phys. Rev. Lett.* **70** 3279
[13] Dhar D and Thomas P B 1993 *Europhys. Lett.* **21** 965
[14] See for example, Rado G T, Wright R W and Emerson W H 1950 *Phys. Rev.* **80** 273
[15] Kumar D and Dattagupta S 1983 *J. Phys.: Condens. Matter* **16** 3779
[16] Gunton J D and Droz M 1983 *Introduction to the Theory of Metastable and Unstable States* (Berlin: Springer) pp 49–92
[17] Binder K and Stoll E 1973 *Phys. Rev. Lett.* **31** 47
Binder K and Stauffer D 1976 *Adv. Phys.* **25** 345
Stauffer D, Coniglio A and Heermann D W 1982 *Phys. Rev. Lett.* **49** 1299
Stauffer D 1992 *Int. J. Mod. Phys.* C **3** 1059
[18] Obukhov S P 1990 *Phys. Rev. Lett.* **65** 1395
Dhar D and Majumdar S N 1990 *J. Phys. A: Math. Gen.* **23** 4333